# *Design and Analysis of a Low Reynolds Number Airfoil*

Prepared for Dr. J. McCuan

MATH 6514: Industrial Math

Presented by Nick Borer

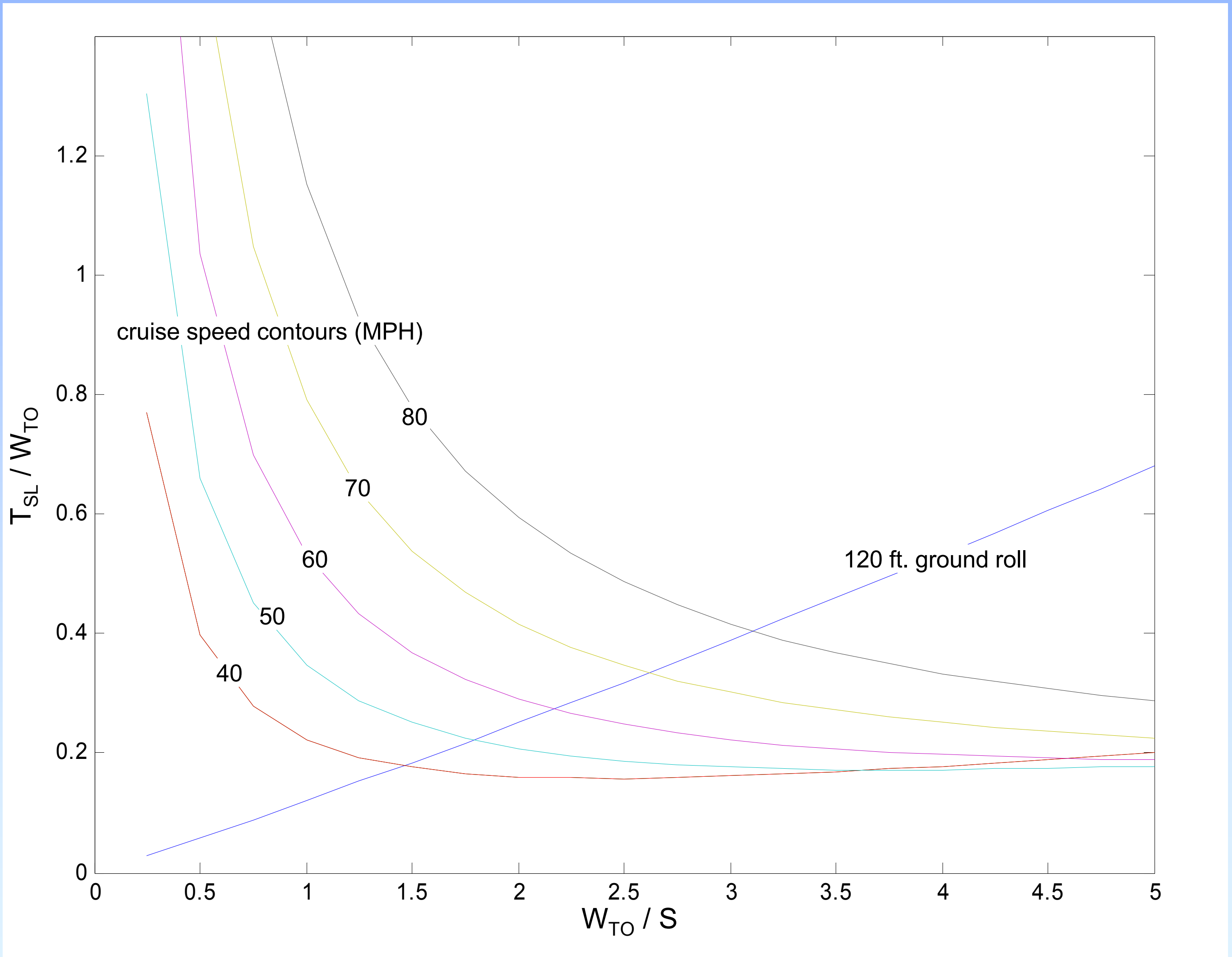
# Airfoil Design

- In the past, airfoils were designed experimentally and catalogued for future use
- The advent of the digital computer has facilitated custom airfoil design for a given wing planform
- There are several approaches to custom airfoil design
  - Trial and error
  - Optimization methods (automated trial and error)
  - Inverse methods
- My work focuses on the optimization method because I am very familiar with optimization techniques

# Design Criteria

- The application in mind is for a low-Reynolds number airfoil that will operate on a flying wing UAV
- Reynolds numbers will range between 200,000 and 700,000 for level flight
  - Airfoil should be designed to operate well between 100,000 and 1,000,000
- This said, the actual viscous calculations do not appear in the design process!
  - Viscous effects calculated after the design process
  - Pressure distributions chosen via heuristics for "good" low-Reynolds number design

# Design Plot for UAV



# *Approach*

- Determine airfoil geometry from input pressure distribution via incompressible, inviscid analyses
  – Ideal application for the vortex panel method
  – Although assumed incompressible, moderate amounts of compressibility can be predicted via Prandtl-Glauert or Karmen-Tsien compressibility corrections (stretch of geometry in x-direction)
- Compare inviscid results to viscous results post-design
- Three analysis routines tested
  – Custom vortex-panel code written in Matlab
  – XFOIL (inviscid only; used as benchmark)
  – XFOIL (viscous; vortex-panel method with boundary layer analysis)

# Vortex Panel Method: Theory

- The vortex panel method belongs to a more general class of analyses known as panel methods
  - All panel methods rely on a superposition of elementary flows in potential (incompressible, inviscid) flow to solve a given problem
  - "Vortex" panel method implies the use of vortex and uniform flows to solve the problem
- It all starts with the 2D incompressible continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

# Vortex Panel Method: Theory

- Stream function (flow abstraction)

$$\frac{\partial \Psi}{\partial y} = u; -\frac{\partial \Psi}{\partial x} = v$$

- Into continuity, get Laplace's Equation

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = 0$$

- Elemenary solution to vortex and uniform flow

$$\Psi_{vortex} = \frac{\Gamma}{2\pi} \ln(r)$$

$$\Psi_{uniform} = V_{\infty} y$$

# Vortex Panel Method: Theory

- Break into components along a streamline to get

$$\Psi = u_\infty y - v_\infty x - \frac{1}{2\pi}\oint \gamma_0 \ln(|r - r_0|)ds_0 - C = 0$$

- Evaluated over *n* segments (panels), this becomes

$$u_\infty y_i - v_\infty x_i - \sum_{j=1}^{n} \frac{\gamma_{0,j}}{2\pi} \int_j \ln(|r - r_0|)ds_0 - C = 0$$

- The integral in the middle can be evaluated analytically, and together are known as the aerodynamic influence coefficients

$$A_{i,j} = \frac{1}{2\pi}\int \ln(|r - r_0|)ds_0$$

# Vortex Panel Method: Theory

- Now we have *n* equations and *n*+1 unknowns, so we add in the Kutta condition

$$\gamma_{0_{TE-upper}} = -\gamma_{0_{TE-lower}}$$

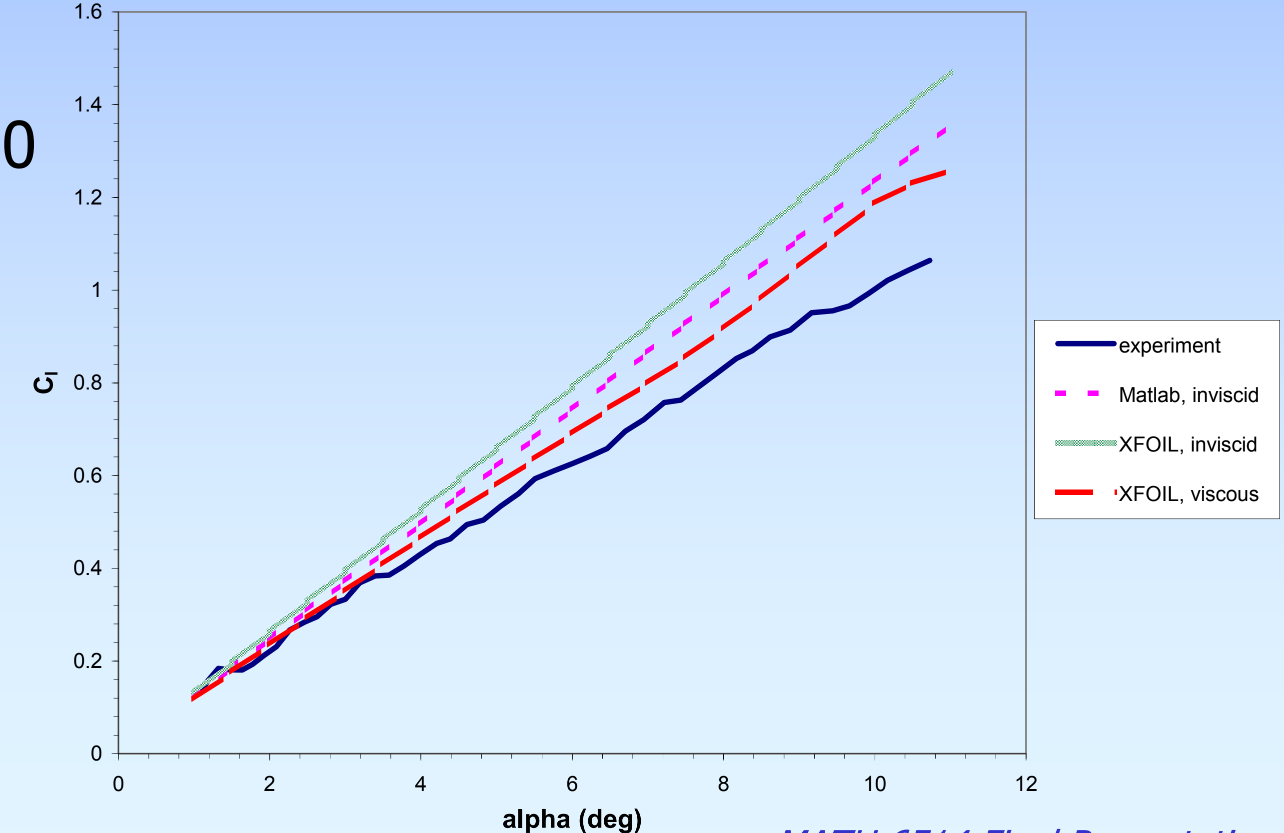
- Finally, we have a system of *n*+1 equations and *n*+1 unknowns that can be easily inverted and solved

$$u_\infty y_i - v_\infty x_i - \sum_{j=1}^{n} A_{i,j}\gamma_{0,j} - C = 0$$

$$\gamma_{0,1} + \gamma_{0,n} = 0$$

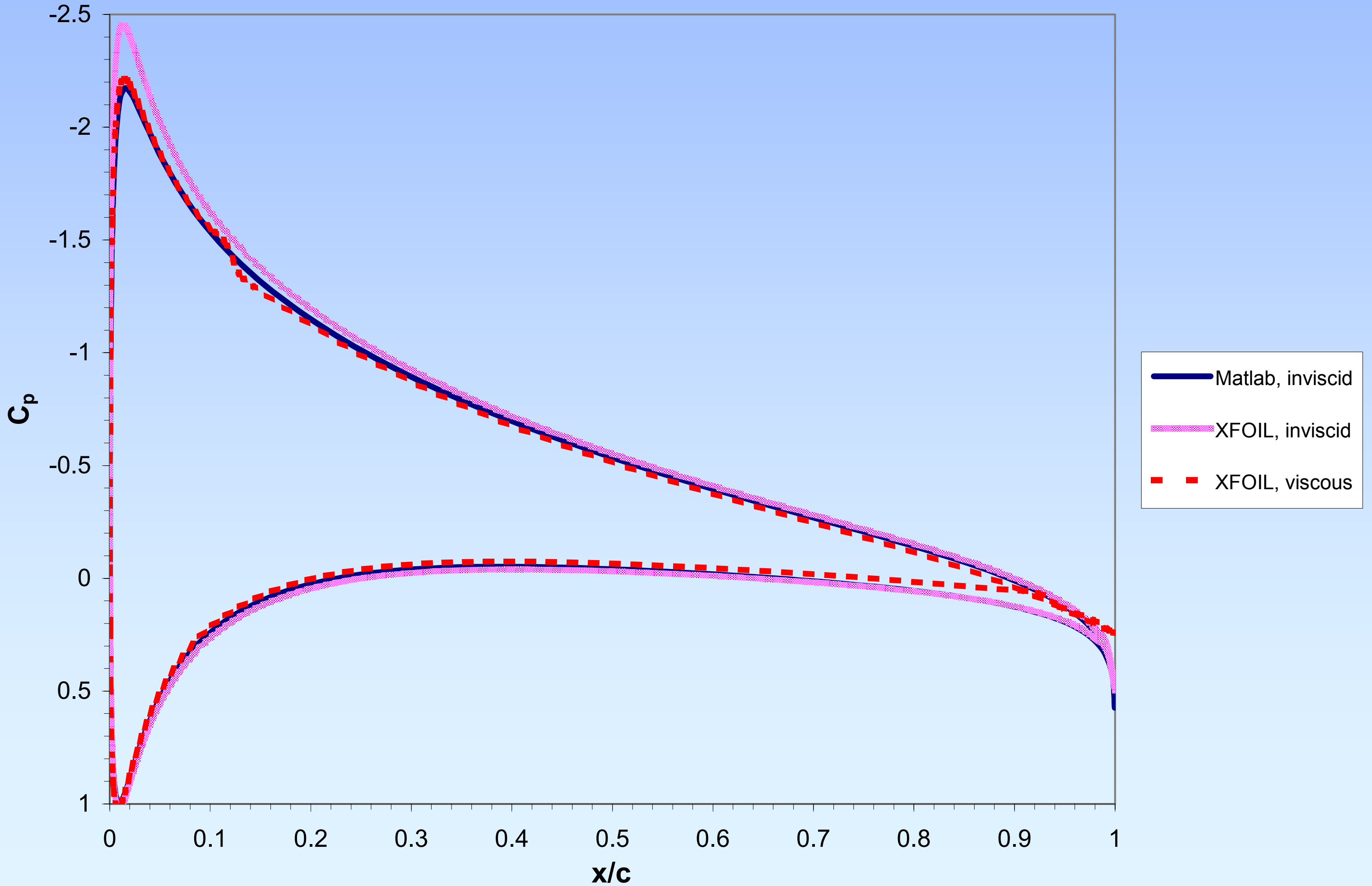
# Validation of Panel Code

- All three panel codes (Matlab, XFOIL-inviscid, and XFOIL-viscous) were compared against trusted experimental data for a NACA 0015 airfoil
- Conditions (when applicable):
  - $\alpha = 5^\circ$
  - Re = 1,950,000
  - M = 0.29



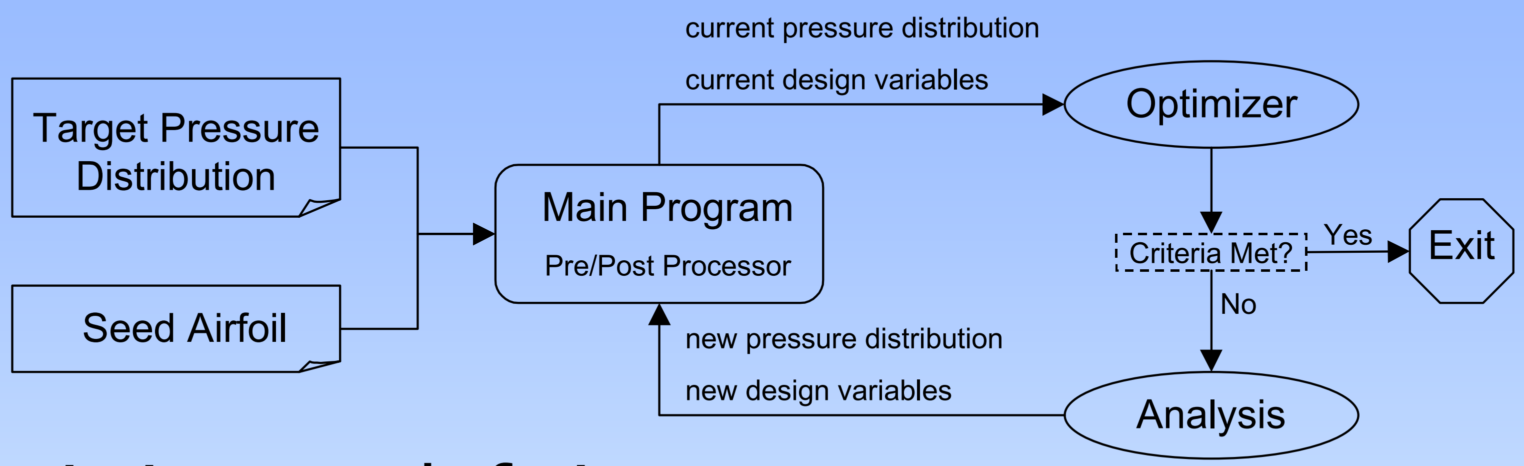
# Pressure Distribution Comparison

- All three methods yielded similar results for output pressure distribution



# Optimization Setup

- Design method

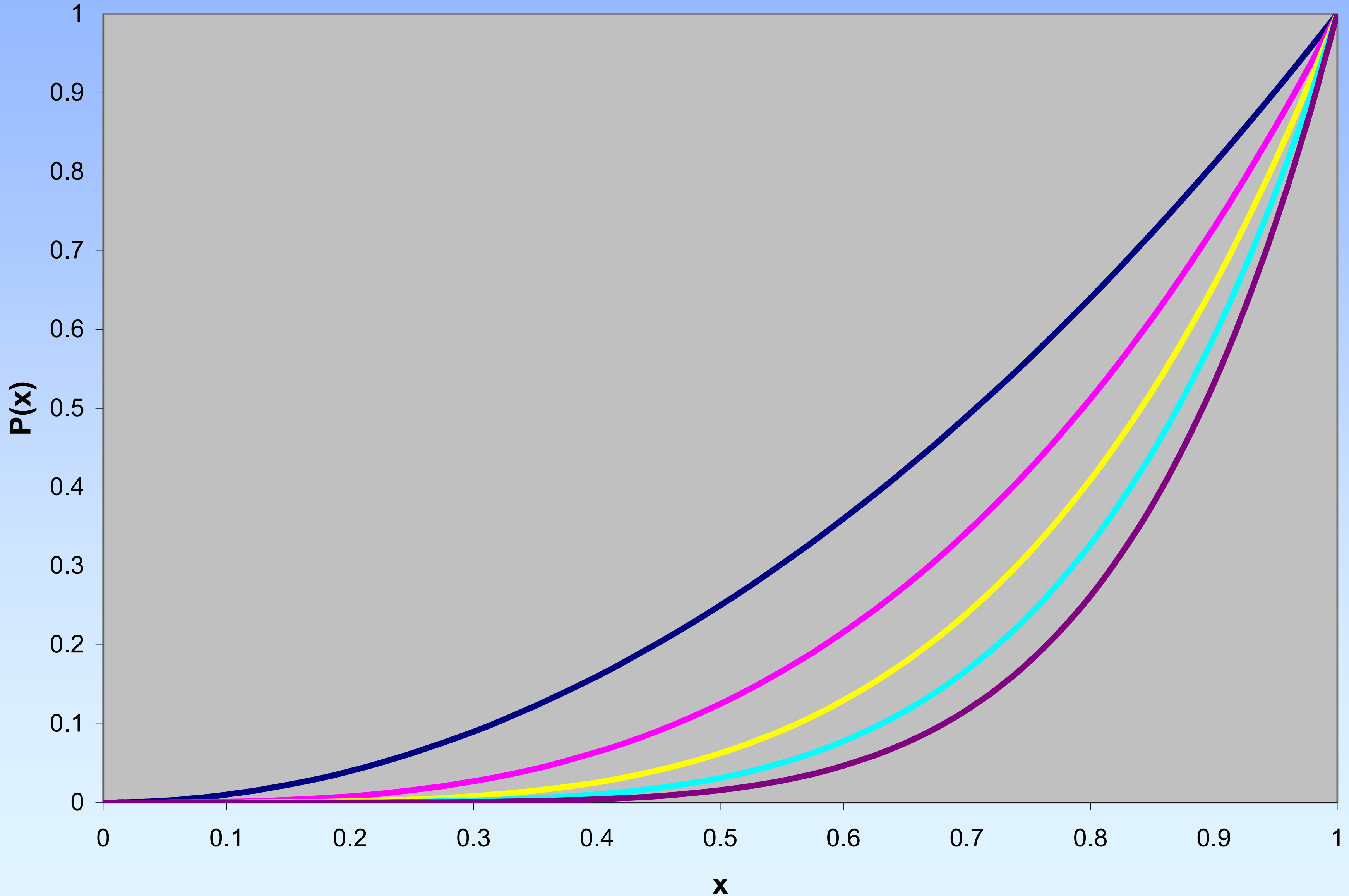



- Optimizer used: fmincon
  - Sequential Quadratic Programming (SQP) optimizer
  - capable of handling nonlinear constraints
- Design variables consist of:
  - multipliers to bump functions (coefficients)
  - upper and lower airfoil scaling factors
  - design angle of attack (incidence)

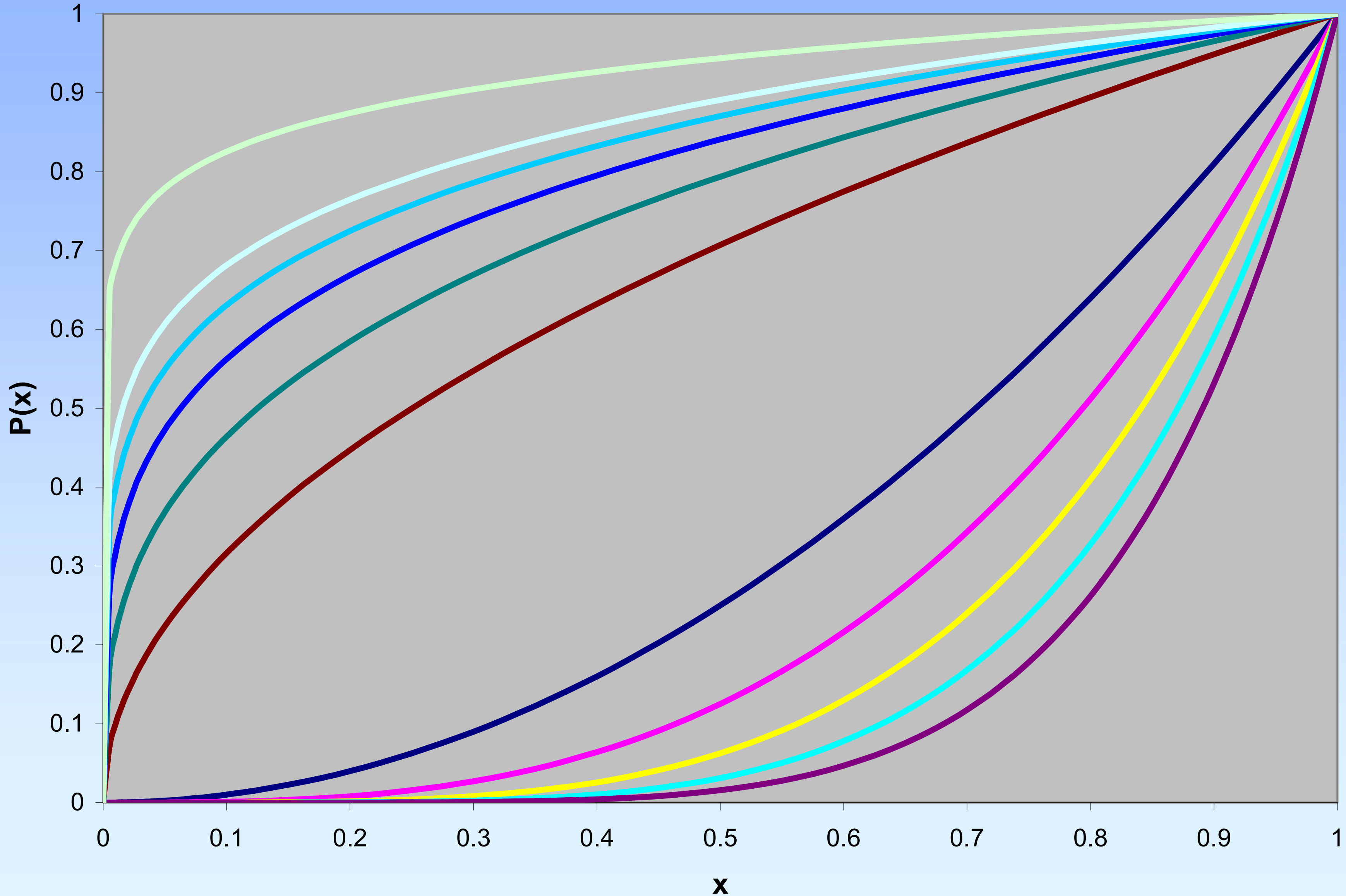
# Bump Function Selection

- There are too many airfoil coordinates to consider each y-ordinate as an independent variable
- The geometry can be controlled through the use of bump functions
- Four series of bump functions tested:
  - Sixth-order polynomial (poly1)
  - Sixth-order polynomial plus inverse (poly2)
  - Hybrid polynomial with centering (poly3)
  - Hicks-Henne functions (hicks)
- Each of these modified the airfoil geometry in different ways

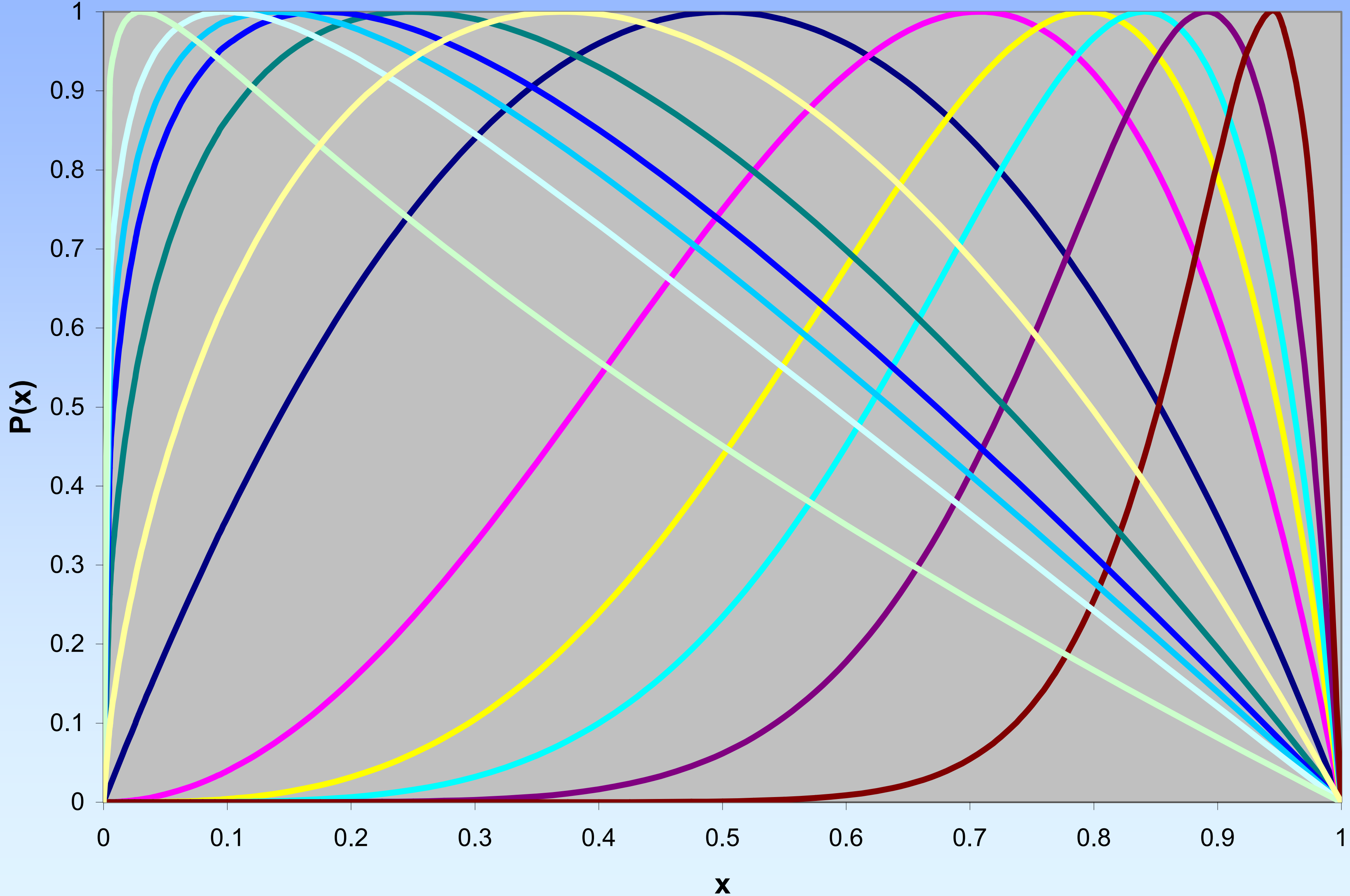
# P(x) = poly1



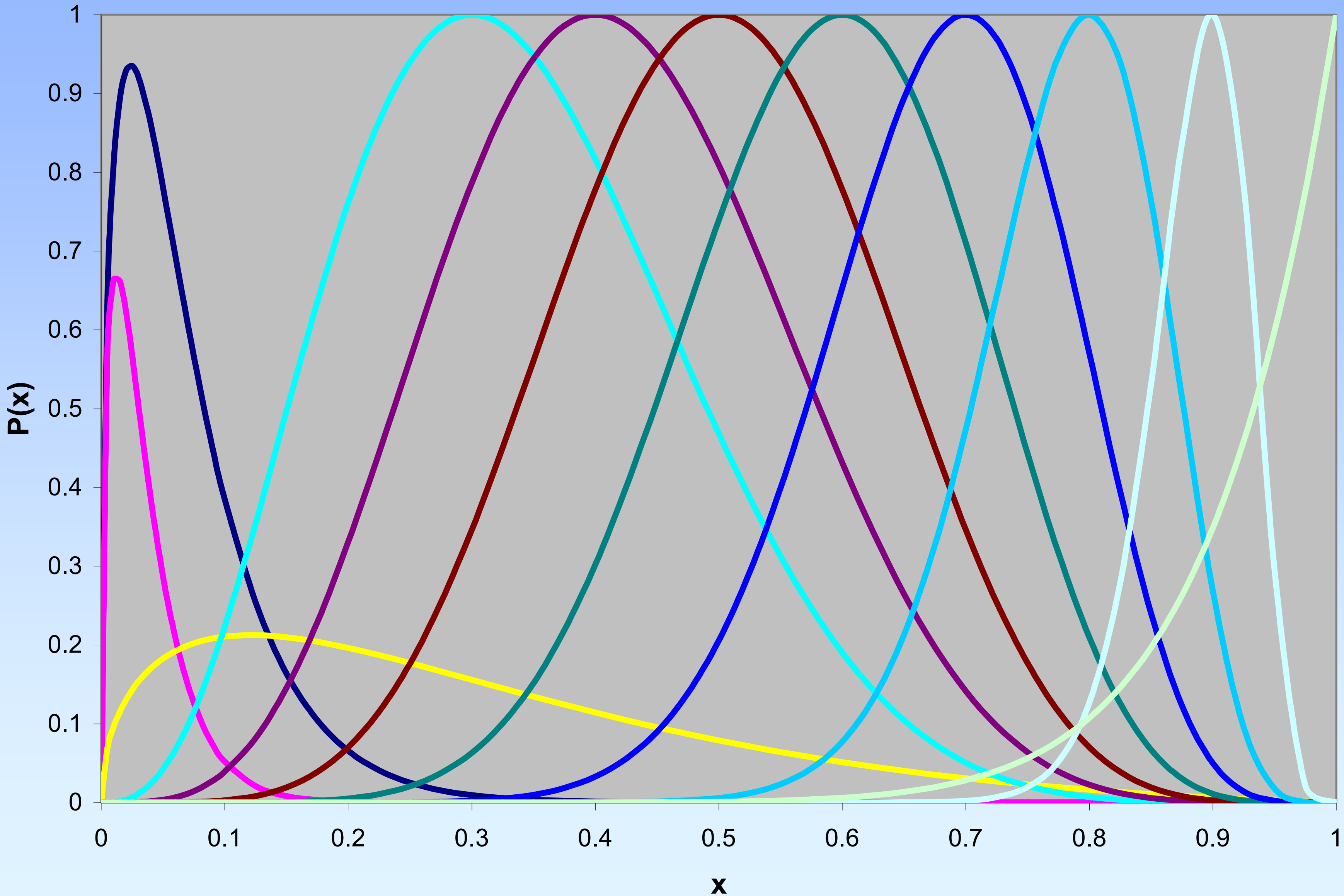
# P(x) = poly2



# P(x) = poly3



# P(x) = hicks

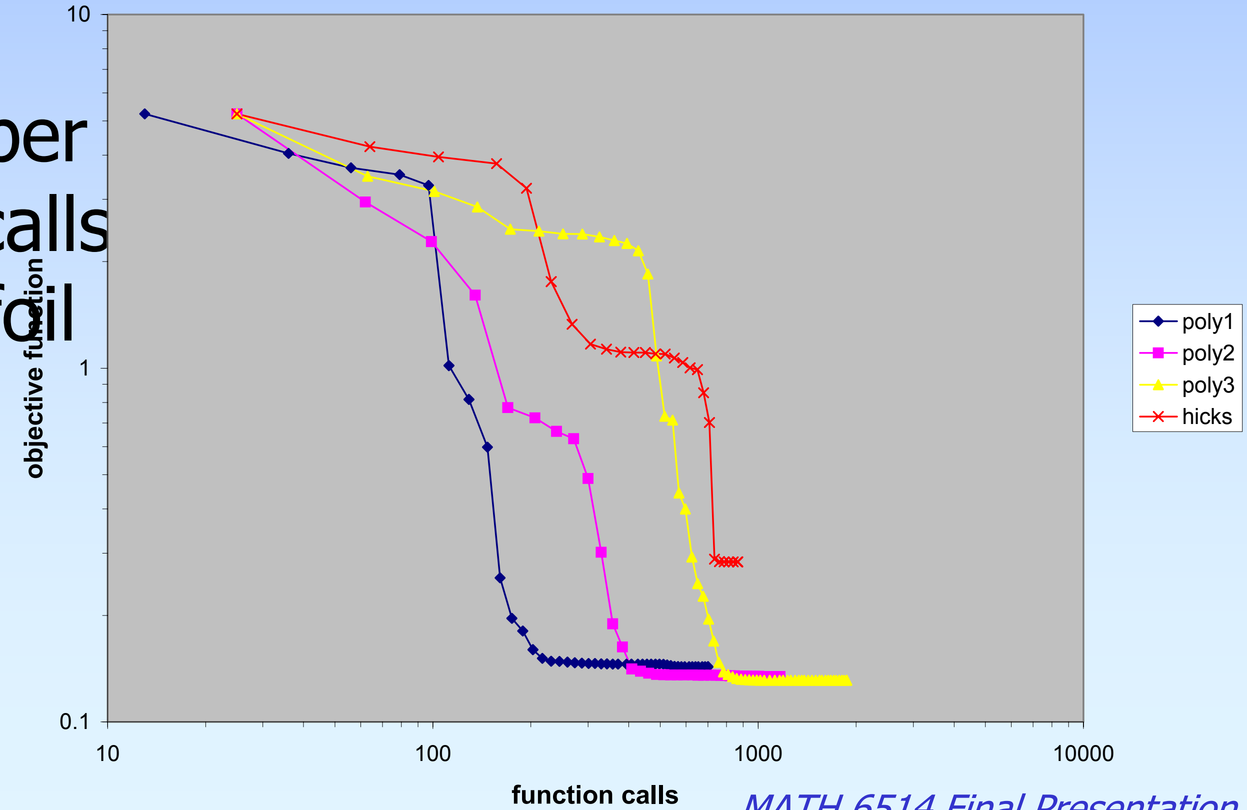


# Design Method Validation

- Wanted to check if the design algorithm would converge to a known airfoil given the appropriate pressure distribution
  - NACA 4412 airfoil at $\alpha = 0°$, M = 0.5
  - Pressure distribution generated from Matlab panel code to eliminate experimental noise
- Started with two seed airfoils of different families
  - NACA 0012 (symmetric, same thickness form as 4412)
  - NACA 23015 (cambered, different thickness form)
- This also provided a way to see which bump function was the most efficient and which was the most robust

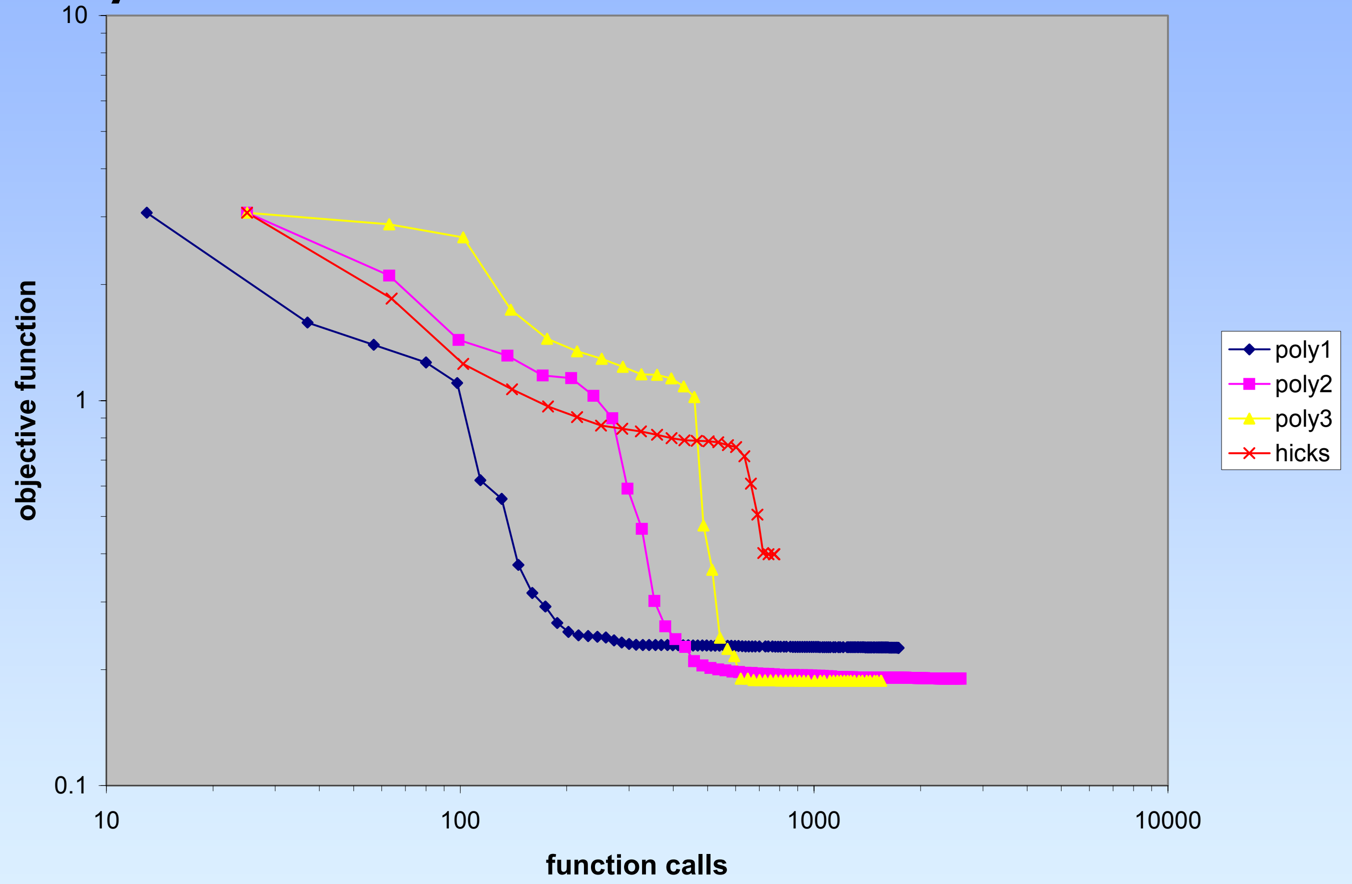
# Design Validation: Results

- Validation found some bugs in the code
- All three polynomial bump functions converged to the correct geometry
- The Hicks-Henne functions did not converge properly
- Efficiency: number of function calls for 0012 airfoil



# Design Validation: Results

- Efficiency for 23015 seed airfoil



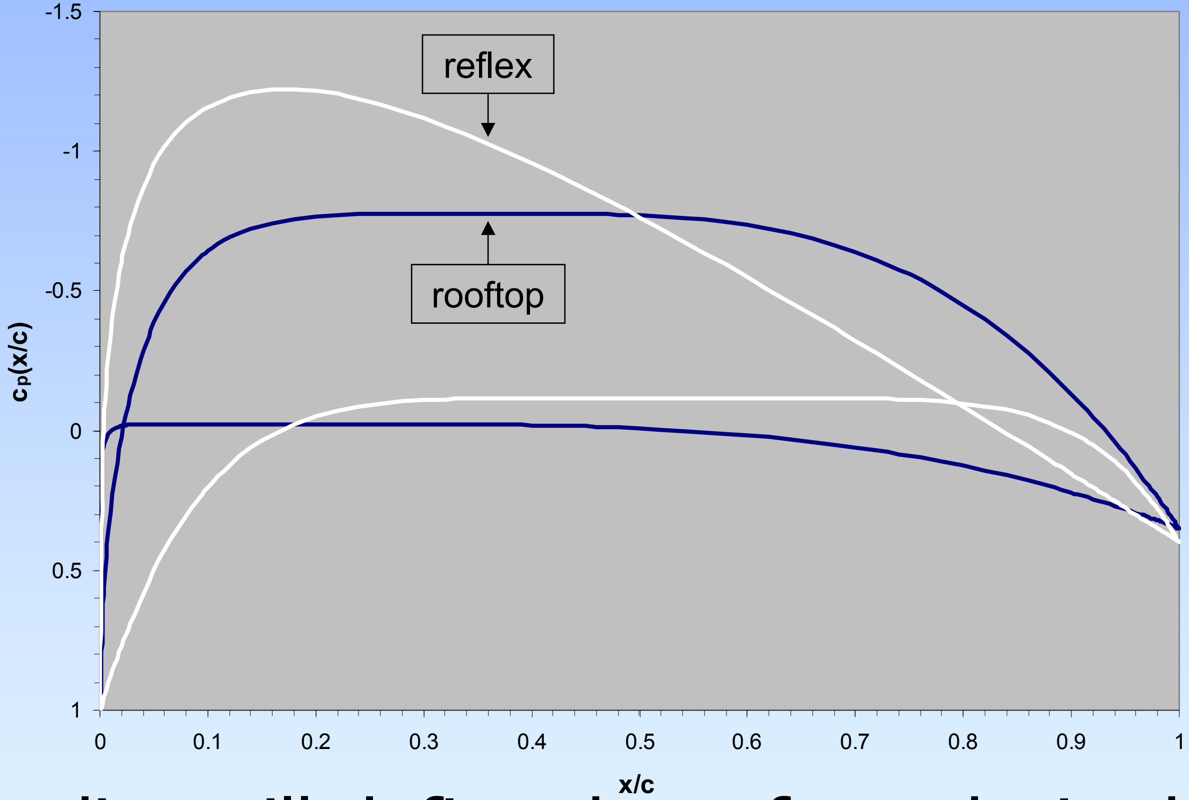

- These results point to poly3 as the most robust set of bump functions

# Design Studies

- Created two input pressure distributions for this speed regime
- Laminar flow: "rooftop" pressure distribution delays adverse pressure gradient, thus delaying pressure-induced transition
  - Unfortunately, this typically results in a highly rear-loaded airfoil, which can increase trim drag
- Flying wing: "reflex" pressure distribution used to trim out nose-down pitching moment at design conditions
  - This necessitates large changes in pressure on both surfaces of the airfoil, thus making laminar flow difficult to achieve

# *Input Pressure Distributions*

- Defined via heuristics and a generic polynomial to ensure that the distributions were smooth



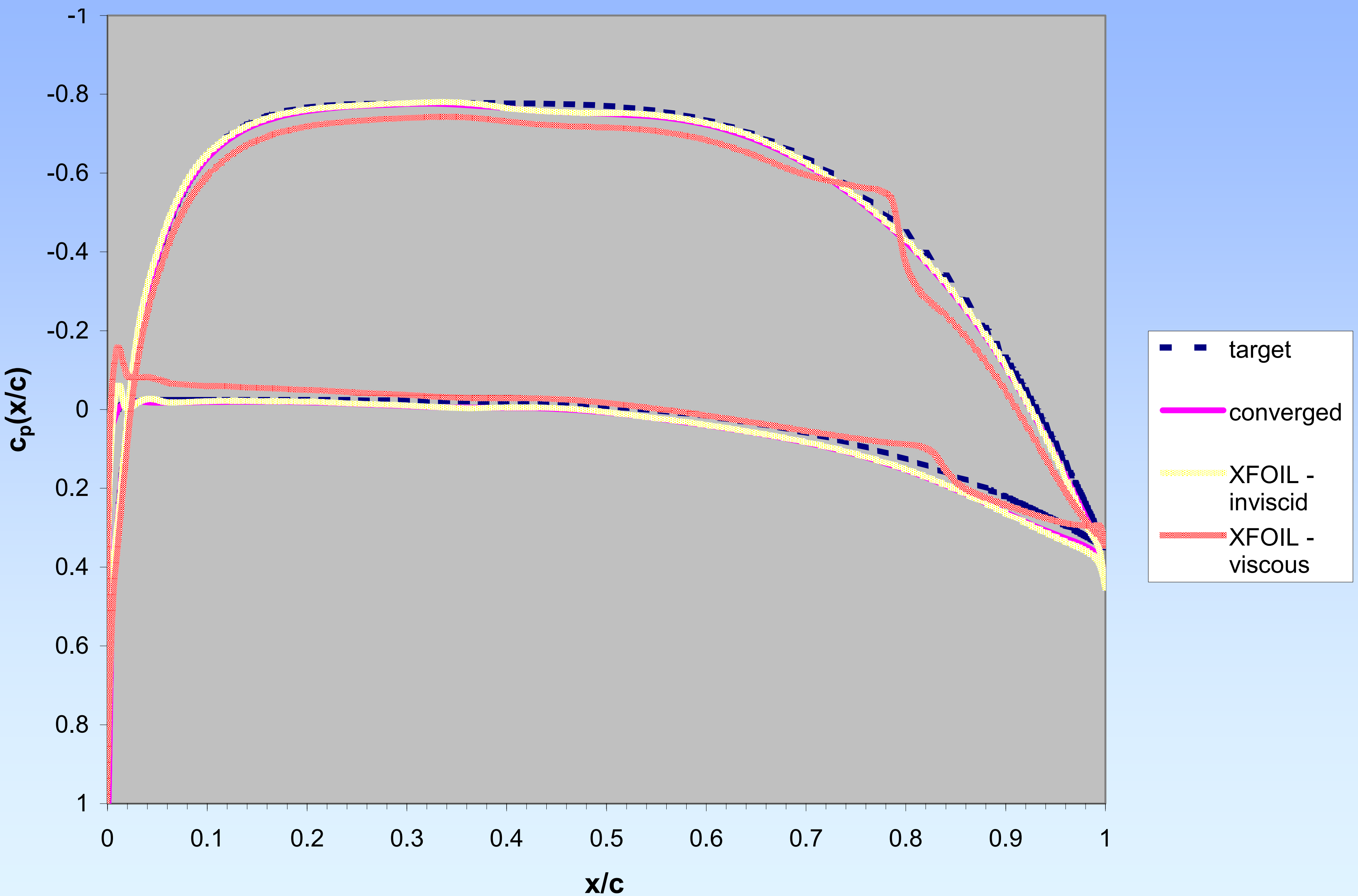

- Later studies will define these from desired wing lift distributions: elliptic spanwise, reflex chordwise, etc.

# *Results*

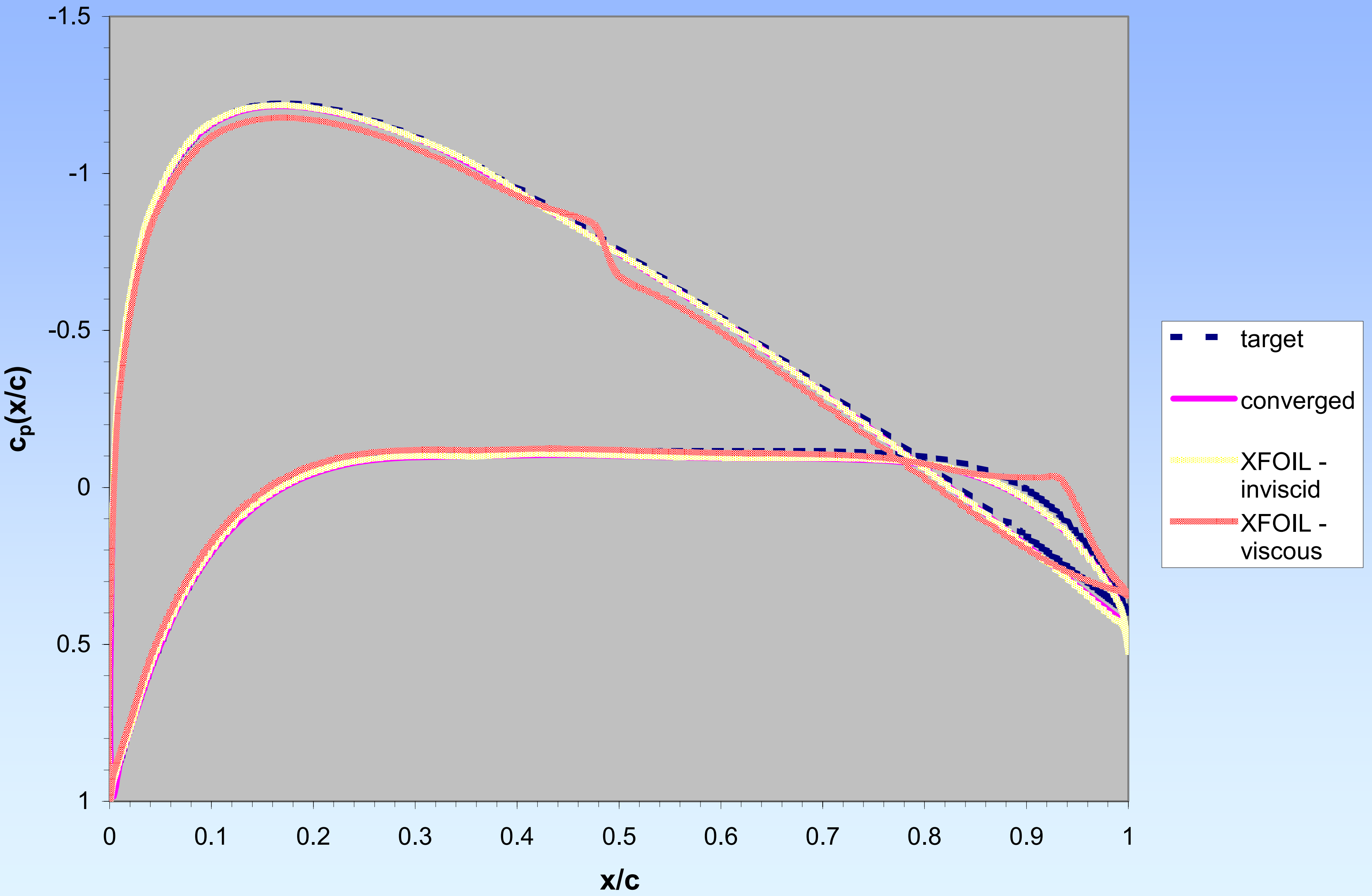
- Both cases converged, and the results were compared against the XFOIL solutions

| | | *Matlab* | | *XFOIL* | |
|---|---|---|---|---|---|
| | *parameter* | *target* | *converged* | *inviscid* | *viscous* |
| rooftop | $C_l$ | 0.6000 | 0.6066 | 0.6093 | 0.5322 |
| | $C_m$ | * | -0.1387 | -0.1394 | -0.1225 |
| | $C_d$ | 0.0000 | 0.0008 | -0.0003 | 0.0055 |
| reflex | $C_l$ | 0.6000 | 0.6169 | 0.6153 | 0.5637 |
| | $C_m$ | 0.0000 | -0.0043 | -0.0078 | 0.0027 |
| | $C_d$ | 0.0000 | 0.0008 | -0.0002 | 0.0081 |

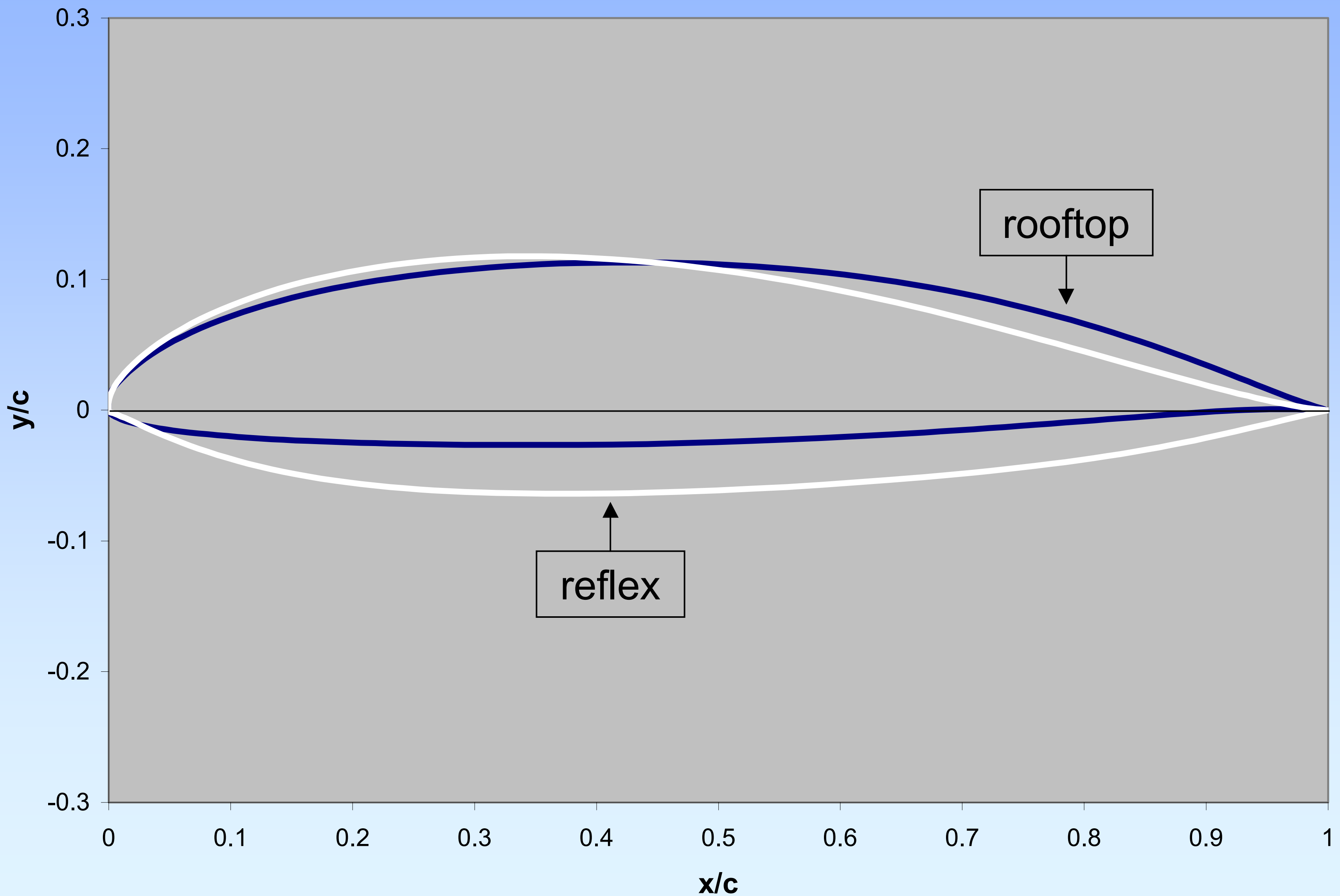
# Pressure Distribution: Laminar Flow Airfoil



# Pressure Distribution: Reflex Airfoil



# Converged Airfoil Shapes

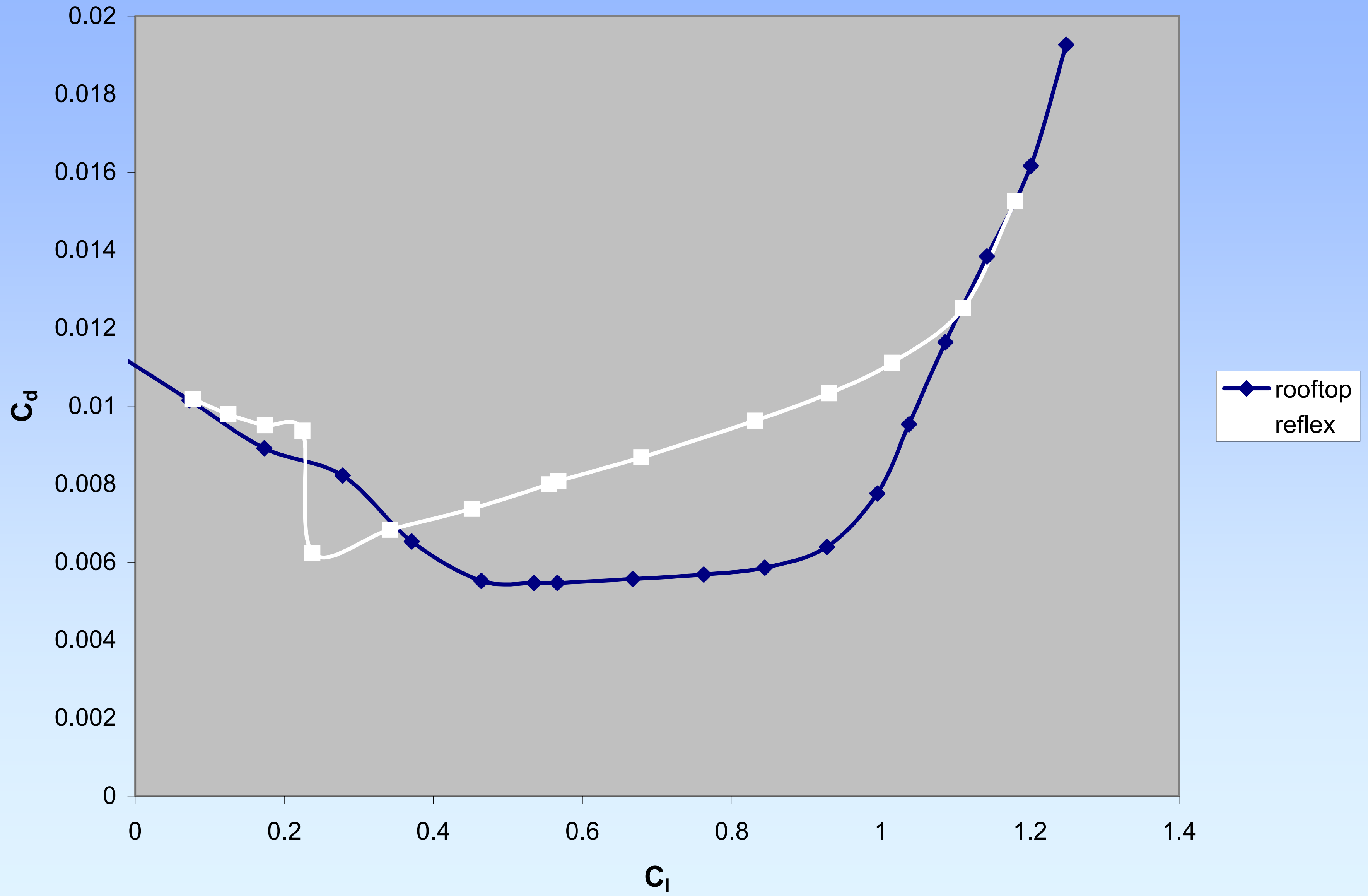


# Comments and Future Work

- The airfoils that resulted from this design code reflected the physics of the design problem
  - The laminar airfoil has a relatively small leading edge radius and even thickness form
  - The reflex airfoil has noticeable positive camber near the leading edge and reflex camper near the trailing edge
- Design method would be better if it could be coupled with a boundary layer analysis
  - Panel code geometry could update with boundary layer displacement thickness
- This 2D design tool can be coupled with a 3D analysis for efficient wing design

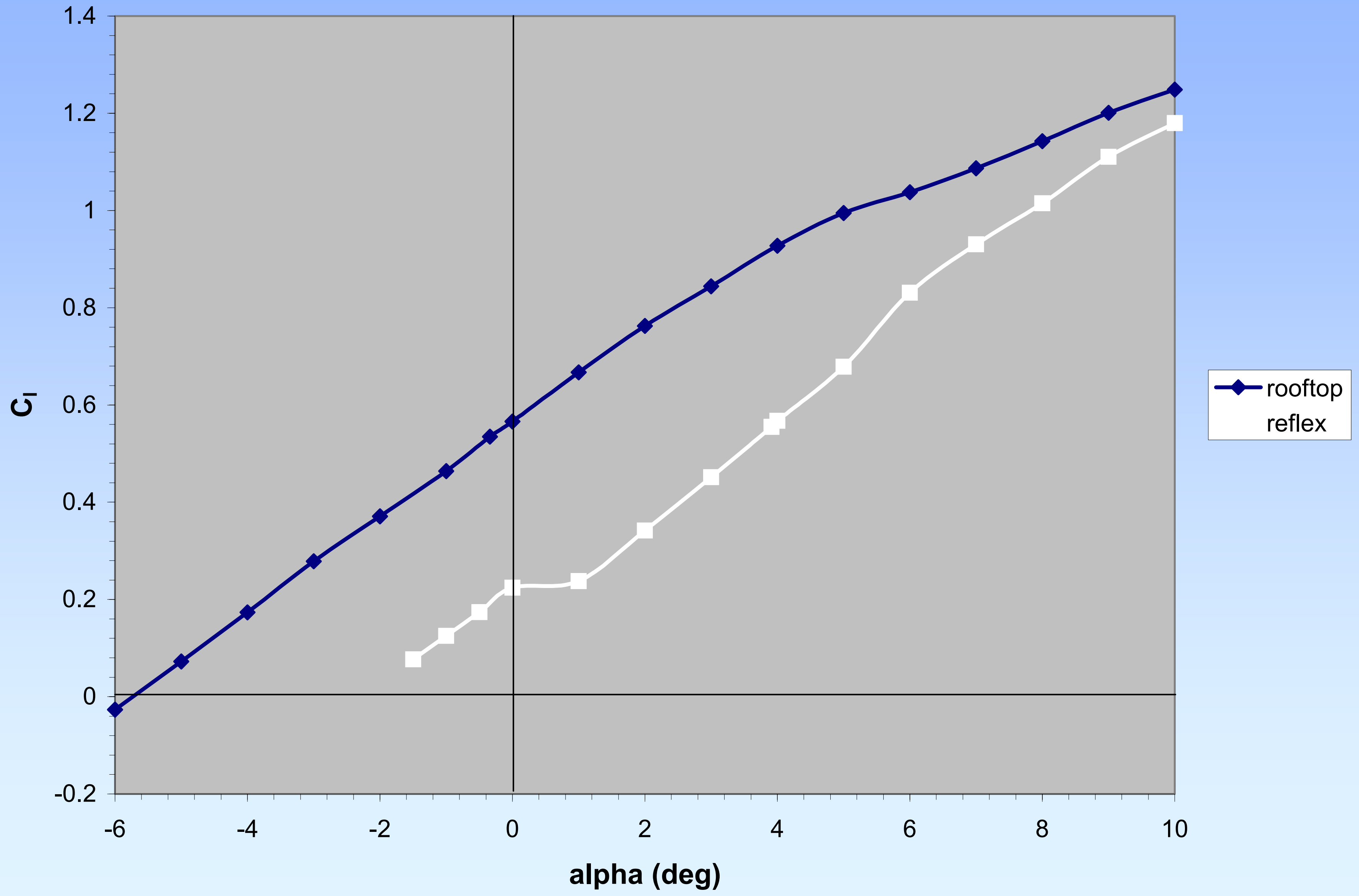
# Backups

# Drag Polars



0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
$C_d$
$C_l$
rooftop
reflex

# Lift Curves



# Moment Curves